“From Market Making to Matchmaking: Does Bank Regulation Harm Market Liquidity?” Internet Appendix

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Contents

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1 Analysis with General Private Value Distribution $G(\cdot)$

Section 3 in the paper presents a simplified version of our model in which we assume that customers' private value follows a uniform distribution. In this section of the Internet Appendix we provide the propositions and proofs for the general setup of the model as described in Section 2 of the paper. Specifically, we assume that the private value x is a random variable defined on $x \in [0, \infty)$, with cumulative distribution function G. The only structure we impose on the distribution of private values is the monotone-hazard-rate assumption. We state this assumption in terms of the inverse hazard function (or Mills ratio) of G,

$$\zeta(x) = \frac{1 - G(x)}{G'(x)},$$

and specify that $\zeta(x)$ is non-increasing in x.

As in the paper, we first characterize the equilibria that exist in different regions of c_B.[1] We then ask what happens to overall customer welfare and market outcomes when c_B goes up.

1.1 Existence of Equilibrium when $c_B \leq c_{NB}$

The main case we consider is when the balance sheet cost of the bank dealer is lower than that of the non-bank dealer, $c_B \leq c_{NB}$. This is the natural case to consider given the too-big-to-fail subsidy and the dominance of bank dealers in the corporate bond market. In this case, the bank dealer's problem of maximizing expected profit from providing market-making and matchmaking services is

$$\max_{0 \leq f \leq S \leq c_{NB}} \Pi_B(S, f; c_B) \equiv \frac{2\mu}{r}\left[(\mathcal{H}f - I)(G(b) - G(f)) + (S - c_B)(1 - G(b))\right], \tag{1}$$

where $b = \frac{S - \mathcal{H}f}{1 - \mathcal{H}}$.

Let $\phi(x) \equiv x - \zeta(x) = x - \frac{1 - G(x)}{G'(x)}$.[2] We begin by establishing the existence of an equilibrium and characterizing its structure.

Proposition 1. *When $c_B \leq c_{NB}$ and $I < \mathcal{H}c_B$, the bank dealer operates both market-making and matchmaking services, and the equilibrium is characterized as follows:*

1. *If $\phi(c_{NB}) \leqslant 0$ and $I \in (0, \mathcal{H}c_B)$, there is a* ***constrained bank dealer*** equilibrium *($S^* = c_{NB}$), and f^* is the minimal solution of*

$$f^* = \arg\max_f \frac{2\mu}{r}\left[(\mathcal{H}f - I)\left(G\left(\frac{c_{NB} - \mathcal{H}f}{1 - \mathcal{H}}\right) - G(f)\right) + (c_{NB} - c_B)\left(1 - G\left(\frac{c_{NB} - \mathcal{H}f}{1 - \mathcal{H}}\right)\right)\right]. \tag{2}$$

2. *If $\phi(c_{NB}) > 0$, then*

   (a) *If $I \in (0, \mathcal{H}\phi(c_{NB}))$, there exists $\underline{c} \in \left(\frac{I}{\mathcal{H}}, c_{NB}\right)$, such that*

      i. *If $c_B \in \left(\frac{I}{\mathcal{H}}, \underline{c}\right)$, there is an* ***unconstrained bank dealer*** equilibrium *($S^* < c_{NB}$) that satisfies the following conditions:*

$$\phi(f^*) = \frac{I}{\mathcal{H}},\ \phi(b^*) = \frac{c_B - I}{1 - \mathcal{H}},\ S^* = \mathcal{H}f^* + (1 - \mathcal{H})b^*; \tag{3}$$

      ii. *If $c_B \in [\underline{c}, c_{NB}]$, there is a* ***constrained bank dealer*** equilibrium *($S^* = c_{NB}$), and f^* is the minimal solution of (2).*

   (b) *If $I \in [\mathcal{H}\phi(c_{NB}), \mathcal{H}c_{NB}]$ and $c_B \in \left(\frac{I}{\mathcal{H}}, c_{NB}\right]$, there is a* ***constrained bank dealer*** equilibrium *($S^* = c_{NB}$), and f^* is the minimal solution of (2).*

All proofs are provided at the end of this Internet Appendix. The proof of the above proposition can be found in A.1.

[1] If there exist multiple Nash equilibria, then we choose the equilibrium that generates the highest overall customer welfare; if there are multiple equilibria that generate the same customer welfare, then we randomly choose one of them.

[2] In the mechanism design literature, $\phi(x)$ is sometimes called the virtual valuation function.

1.2 Existence of Equilibrium when $c_B > c_{NB}$

It is clear that in this case, only the non-bank dealer provides market-making services, and we focus only on equilibria in which the bank dealer engages in matchmaking. Specifically, given the bank dealer's choice of f, the non-bank dealer's problem is

$$\pi_{NB}(c_B) = \max_{c_{NB} \leq S \leq c_B} \Pi_{NB}(S) = \frac{2\mu}{r}\left[(S - c_{NB})(1 - G(b))\right].$$

We impose a tie-breaking rule that the bank dealer does not offer the market-making service if his profit from it would be zero. This implies that when $S_{NB} = c_B$, the bank dealer does not operate the market-making business.

Given the non-bank dealer's choice of S, the bank dealer's problem is

$$\pi_B(c_B) = \max_{0 \leq f \leq S} \Pi_B(f) = \frac{2\mu}{r}(\mathcal{H}f - I)(G(b) - G(f)),$$

where $b = \frac{S - \mathcal{H}f}{1-\mathcal{H}}$. The following proposition establishes the existence of an equilibrium when the balance sheet cost of the bank dealer exceeds that of the non-bank dealer.

Proposition 2. *When $c_B > c_{NB}$ and $I < \mathcal{H}\min\{\tilde{c}_B, c_B\}$, there exists a unique equilibrium such that the non-bank dealer operates the market-making service and the bank dealer operates the matchmaking service, with $\tilde{c}_B$ as the unique solution of*

$$\xi(\tilde{c}_B) - \frac{\tilde{c}_B - c_{NB}}{1 - \mathcal{H}} = 0,$$

provided that G is concave or G is convex with $G''' < 0$ and $\mathcal{H} < \frac{1}{2}$. In particular, there exists $\bar{c} > \max\{c_{NB}, \frac{I}{\mathcal{H}}\}$ such that,

1. *If $c_B \in (c_{NB}, \bar{c})$, the equilibrium is a **constrained non-bank dealer** equilibrium with $S^* = c_B$;*
2. *If $c_B \in (\bar{c}, \infty)$, the equilibrium is an **unconstrained non-bank dealer** equilibrium with $S^* < c_B$.*

The proof is provided in A.2.

1.3 What Happens when Bank Regulatory Costs Increase?

While in the paper we collect all relevant comparative statics in Proposition 2, in this section of the Internet Appendix we provide separate propositions for each equilibrium region.

Proposition 3. *When c_B increases in the **constrained bank dealer** equilibrium,*

1. *The spread is unchanged ($S^* = c_{NB}$), the matchmaking fee f^* decreases, and average transaction costs decrease;*
2. *Trading volume increases, matchmaking increases, and market making decreases;*
3. *Overall customer welfare, π_c, increases.*

The proof is provided in A.3.

Proposition 4. *When c_B increases in the **unconstrained bank dealer** equilibrium,*

1. *The spread S^* increases, the matchmaking fee f^* is unchanged, and average transaction costs increase if $c_B < (1-\mathcal{H})f^* + I$ and decrease if $c_B \geqslant (1-\mathcal{H})f^* + I$;*
2. *Trading volume is unchanged, matchmaking increases, and market making decreases;*
3. *Overall customer welfare, π_c, decreases.*

The proof is provided in A.4.

Proposition 5. *When c_B increases in the **constrained non-bank dealer** equilibrium,*

1. *$S^* = c_B$ increases, f^* increases, and the change in average transaction costs is ambiguous;*
2. *Trading volume decreases, market making decreases, and the change in matchmaking is ambiguous;*
3. *Overall customer welfare, π_c, decreases.*

The proof is provided in A.5.

1.4 What Happens when the Cost of Matchmaking Declines?

In Section 6 at the end of the paper we mention another important development that took place in the past decade: technological advances that reduced the cost of matchmaking. This development is represented in our model by a reduction in the cost of search in the matchmaking mechanism, I. While one might assume that a reduction in the cost of search would always increase overall customer welfare, the propositions below show that such an unambiguous result can only be shown in the constrained equilibria.

Proposition 6. *When I decreases in either the* ***constrained bank dealer*** *equilibrium (section 1.1) or the* ***constrained non-bank dealer*** *equilibrium (section 1.2),*

1. *S^* is unchanged, f^* decreases, and average transaction costs decrease;*
2. *Trading volume increases, matchmaking increases, and market making decreases;*
3. *Overall customer welfare, π_c, increases.*

The proof is provided in A.6.

Proposition 7. *When I decreases in the* ***unconstrained bank dealer*** *equilibrium (section 1.1),*

1. *S^* increases (decreases) if ζ is convex (concave), f^* decreases, and the change in average transaction costs is ambiguous;*
2. *Trading volume increases, matchmaking increases, and market making decreases;*
3. *The change in overall customer welfare, π_c, is ambiguous if ζ is convex, and is positive if ζ is concave.*

The proof is provided in A.7.

2 Benevolent Bank Dealer

Why does customer welfare increase in the constrained bank dealer equilibrium? Competition from the non-bank dealer prevents the bank dealer from increasing the spread as regulatory costs rise, decreasing the rents he collects from the market-making business and incentivizing a shift into the matchmaking business. The competition element is important because the bank dealer wields market power in his interaction with customers, and therefore he prices the market-making spread to extract economic rents. As we mention in the paper, it has long been established that dealers in the over-the-counter corporate bond market have market power. This market power is critical to delivering our results.

To illustrate this point, in this section we discuss a variation of our model in which the bank dealer is benevolent. Specifically, the bank dealer maximizes overall customer welfare subject to the constraint that he is making zero profit on providing liquidity. We can think about the benevolent dealer as arising under a competitive dealer market with free entry. This variation therefore eliminates monopoly rents. We focus on the parameter range $c_B < c_{NB}$, where the bank dealer is the sole liquidity provider, because our goal is to scrutinize the result that customer welfare increases. The bank dealer's problem in this variation of the model is therefore

$$V(c_B) = \max_{0 \le f \le S \le c_{NB}; b \le A} \pi_c = \frac{2\pi}{r} \int_f^b (x - f)\,\mathcal{H}\frac{dx}{A} + \int_b^A (x - S)\,\frac{dx}{A}$$

such that

$$b = \frac{S - \mathcal{H}f}{1 - \mathcal{H}}$$

and

$$\pi_B = \frac{2\pi}{r}\left[\mathcal{H}f\frac{b - f}{A} + (S - c_B)\left(\frac{A - b}{A}\right)\right] = 0.$$

Denote by $(S^*(c_B), \mathcal{H}^*(c_B), f^*(c_B))$ the optimal solution to the above problem when the balance sheet cost is c_B.

Proposition 8. *$V(c_B)$ is decreasing in c_B when $c_B < c_{NB}$.*

The proof is provided in A.8. The setting with a benevolent bank dealer who maximizes customer welfare and breaks even on liquidity provision delivers a stark result: customer welfare will always decrease when regulatory costs rise. The welfare result we obtain in our main model is therefore attributable to the inefficiency introduced by the market power of over-the-counter bank dealers. Section 4 in the paper further investigates the importance of market power for our welfare result by studying an extension with multiple bank and non-bank dealers.

3 Omitted Proofs in Section 4.1: Non-bank Dealer Matchmaking Service

In Section 4.1 we argue that even if the non-bank dealer's matchmaking service is slower than the one operated by the bank dealer ($\mathcal{H}_{NB} < \mathcal{H}_B$), competition from the non-bank dealer causes the bank dealer to set a lower matchmaking fee than in the main model. We formalize and prove this result in this section of the Internet Appendix.

Suppose that with parameters $(\mu, r, A, c_{NB}, c_B, \mathcal{H}_B)$, the equilibrium is a constrained bank dealer equilibrium in our main model, and with parameters $(\mu, r, A, c_{NB}, c_B, \mathcal{H}_B, \mathcal{H}_{NB})$, the equilibrium is a constrained equilibrium in our extension with non-bank dealer matchmaking in Section 4.1. Denote by f_B^1 the matchmaking fee in the constrained bank dealer equilibrium in our main model, and denote by f_B^2 the matchmaking fee of the bank dealer in the constrained equilibrium in the extension with non-bank dealer matchmaking. The following proposition shows that in the equilibrium in which the bank dealer's market-making spread is constrained ($S = c_{NB}$), his matchmaking fee in the Section 4.1 extension is lower than the matchmaking fee he sets in our main model.

Proposition 9. $f_B^2 < f_B^1$.

The proof is provided in A.9. As in our main model, overall customer welfare increases because both f_B^* and f_{NB}^* decrease in the constrained equilibrium (i.e., when $S = c_{NB}$). Both the bank and non-bank dealers make positive profits from matchmaking because their products are differentiated. When c_B increases, the profit margin from market making goes down for the bank dealer, and therefore he lowers the matchmaking fee to attract customers to his matchmaking business. The non-bank dealer optimally responds by lowering her matchmaking fee, but less aggressively so. The decrease in both matchmaking fees results in higher overall customer welfare because (i) some market-making customers benefit from switching to matchmaking, (ii) customers switching from one matchmaking service to another do so because they obtain higher utility, and (iii) additional customers who previously did not trade now find it optimal to trade. Adding competition in matchmaking services therefore does not change the essence of our main result that overall customer welfare can improve when bank regulatory costs rise.

4 Omitted Proofs in Section 4.3: Multiple Bank and Non-Bank Dealers

4.1 Derivatives with Respect to the Number of Non-Bank Dealers

In our extension with multiple bank and non-bank dealers, we have the following results in equation (31):

$$\frac{\partial^2 S^\star}{\partial c_B \partial K} < 0, \quad \frac{\partial^2 f^\star}{\partial c_B \partial K} < 0.$$

These two results can be derived directly from the expressions for the market-making spread and matchmaking fee in equilibrium

$$S^* = \frac{\Lambda c_B + B c_{NB} + A}{1 + J + K}$$

and

$$f^* = \frac{-\Lambda c_B \mathcal{H} K + IJ\left(1 + J + K\right) + B c_{NB} \mathcal{H}\left(1 + J\right) + \mathcal{H} A\left(1 + J\right)}{\mathcal{H}\left(1 + J\right)\left(1 + J + K\right)}.$$

The derivatives are

$$\frac{\partial^2 S^*}{\partial c_B \partial K} = \frac{-\Lambda}{\left(1 + J + K\right)^2} < 0$$

and

$$\frac{\partial^2 f^*}{\partial c_B \partial K} = \frac{-\Lambda}{\left(1 + J\right)\left(1 + J + K\right)^2} < 0.$$

4.2 Two Heterogenous Bank Dealers

In this section, we provide detailed analysis on the extension with two heterogenous bank dealers and one non-bank dealer. The non-bank dealer's cost parameter is $\beta = 1$. For bank dealer i, the cost parameter is $\lambda_i = 1 + x_i$ where $x_1 = \delta$ and $x_2 = -\delta$. Assume $\delta \in (0, 1)$. In this case,

$$Q_M = q_M^1 + q_M^2,$$

$$Q_B = q_B^1 + q_B^2$$

and

$$Q_{NB} = q_{NB}.$$

Note that the first order derivative for the non-bank dealer is

$$\begin{aligned}\frac{r}{2\mu}\frac{\partial \Pi_{NB}}{\partial q_{NB}} &= -c_{NB} + \left(-q_{NB} + 1 - Q_{NB} - Q_B - \mathcal{H}Q_M\right)A \\ &= -c_{NB} + \left(-2q_{NB} + 1 - Q_B - \mathcal{H}Q_M\right)A \\ &= \left(S - c_{NB}\right) - q_{NB}A\end{aligned}$$

and the first order derivatives with respect to the quantities of market making and matchmaking for bank dealer i ($i = 1, 2$) are

$$\begin{aligned}\frac{r}{2\mu}\frac{\partial \Pi_B^i}{\partial q_B^i} &= -\left(1 + x_i\right)c_B - \mathcal{H}Aq_M^i - Aq_B^i + A\left(1 - q_{NB} - Q_B - \mathcal{H}Q_M\right) \\ &= -\left(1 + x_i\right)c_B - A\left(\mathcal{H}q_M^i + q_B^i\right) + A\left(1 - q_{NB} - Q_B - \mathcal{H}Q_M\right) \\ &= \left(S - \left(1 + x_i\right)c_B\right) - A\left(\mathcal{H}q_M^i + q_B^i\right)\end{aligned}$$

and

$$\begin{aligned}\frac{r}{2\mu}\frac{\partial \Pi_B^i}{\partial q_M^i} &= -\mathcal{H}Aq_M^i + \left(\mathcal{H}A\left(1 - q_{NB} - Q_B - Q_M\right) - I\right) - \mathcal{H}Aq_B^i \\ &= \left(\mathcal{H}f - I\right) - \mathcal{H}A\left(q_M^i + q_B^i\right).\end{aligned}$$

When the two bank dealers operate both market-making and matchmaking services, we have the following equilibrium

$$S_{12} = \frac{2c_B + c_{NB} + A}{4}$$

and

$$f_{12} = \frac{-2\mathcal{H}c_B + 3\mathcal{H}c_{NB} + 8I + 3\mathcal{H}A}{12\mathcal{H}}.$$

This is an equilibrium if and only if

$$S_{12} > c_{NB}$$

and

$$S_{12} - \lambda_i c_B > \left(\mathcal{H}f_{12} - I\right) > \mathcal{H}\left(S_{12} - \lambda_i c_B\right)$$

for $i = 1, 2$. We can transform the expressions for the market-making spread and the matchmaking fee above to obtain,

$$S_{12} - \left(1 + x_i\right)c_B = \frac{c_{NB} + A}{4} - \left(\frac{1}{2} + x_i\right)c_B$$

$$\mathcal{H}f_{12} - I = \frac{-2\mathcal{H}c_B + 3\mathcal{H}c_{NB} - 4I + 3\mathcal{H}A}{12}.$$

These expressions can used to simplify the following

$$\left\{\begin{array}{c} S_{12} > c_{NB} \\ S_{12} - \lambda_1 c_B > \left(\mathcal{H}f_{12} - I\right) > \mathcal{H}\left(S_{12} - \lambda_1 c_B\right) \\ S_{12} - \lambda_2 c_B > \left(\mathcal{H}f_{12} - I\right) > \mathcal{H}\left(S_{12} - \lambda_2 c_B\right)\end{array}\right\} \Longleftrightarrow \left\{\begin{array}{c} c_B < \frac{3(1-\mathcal{H})\left(c_{NB}+A\right)+4I}{6-2\mathcal{H}+12\delta} \\ \left(1 - 3\delta\right)\mathcal{H}c_B > I \\ c_B > \frac{3c_{NB}-A}{2}\end{array}\right\} \Longleftrightarrow \left\{\begin{array}{c} \max\left\{\frac{I}{(1-3\delta)\mathcal{H}}, \frac{3c_{NB}-A}{2}\right\} < c_B < \frac{3(1-\mathcal{H})\left(c_{NB}+A\right)+4I}{6-2\mathcal{H}+12\delta} \\ \delta < \frac{1}{3}\end{array}\right\}.$$

This equilibrium, where both bank dealers operate both services, is similar to the case described in Proposition 8 in the paper, and therefore Cutoff 1 above which overall customer welfare increases as c_B goes up can be obtained from equation (30) with $J = 2$:

$$c_B > \frac{16I + \left(1 - \mathcal{H}\right)\left[6A - 2c_{NB}\right]}{\left(9 + 7\mathcal{H}\right)}.$$

When c_B increases in this region, if it passes Cutoff 2 defined by the right-hand side of the following inequality,

$$c_B > \frac{3\left(1 - \mathcal{H}\right)\left(c_{NB} + A\right) + 4I}{6 - 2\mathcal{H} + 12\delta},$$

the condition

$$S_{12}-\lambda_1 c_B > (\mathcal{H} f_{12} - I)$$

is violated. This implies that the high-cost bank dealer will stop offering the market-making service. The game enters into a new equilibrium, in which bank dealer 2 operates both services, bank dealer 1 only operates a matchmaking service, and the non-bank dealer only operates a market-making service.

In the new equilibrium, we have

$$S_{new} = \frac{3(1-\delta)c_B + (3-\mathcal{H})c_{NB} + I + (3-\mathcal{H})A}{(9-\mathcal{H})}$$

and

$$f_{new} = \frac{-\mathcal{H}(1-\delta)c_B + 2c_{NB}\mathcal{H} + (6-\mathcal{H})I + 2\mathcal{H}A}{\mathcal{H}(9-\mathcal{H})}.$$

We can show that both $S_{new} = S_{12}$ and $f_{new} = f_{12}$ when $c_B = \frac{3(1-\mathcal{H})(c_{NB}+A)+4I}{6-2\mathcal{H}+12\delta}$, so the equilibrium is continuous at the transition point.

Overall customer welfare can be written as follows

$$\begin{aligned}
\pi_c &= \frac{2\mu}{r}\left[\int_{f^*}^{b^*} \mathcal{H}(x-f^*)\frac{1}{A}dx + \int_{b^*}^{A}(x-S^*)\frac{1}{A}dx\right] \\
&= \frac{\mu}{r}\frac{1}{A}\left[\mathcal{H}(b^*-f^*)^2 + (A-S^*)^2 - (b^*-S^*)^2\right] \\
&= \frac{\mu}{r}\frac{1}{A}\left[\mathcal{H}\frac{(S^*-f^*)^2}{(1-\mathcal{H})^2} + (A-S^*)^2 - \mathcal{H}^2\frac{(S^*-f^*)^2}{(1-\mathcal{H})^2}\right] \\
&= \frac{\mu}{r}\frac{1}{A(1-\mathcal{H})}\left[\mathcal{H}(S^*-f^*)^2 + (1-\mathcal{H})(A-S^*)^2\right].
\end{aligned}$$

We can show that

$$\frac{d\pi_c}{dc_B} > 0$$

is equivalent to

$$\begin{aligned}
c_B &> \frac{c_{NB}(-9+8\mathcal{H}+\mathcal{H}^2) + (21-5\mathcal{H})I + 2(9-11\mathcal{H}+2\mathcal{H}^2)A}{(9+7\mathcal{H})(1-\delta)} \\
&= \frac{(21-5\mathcal{H})I + (1-\mathcal{H})\left[2(9-2\mathcal{H})A - c_{NB}(9+\mathcal{H})\right]}{(9+7\mathcal{H})(1-\delta)},
\end{aligned}$$

where the expression on the right-hand side is Cutoff 3. Similar to our result from the paper, therefore, overall customer welfare increases if and only if c_B is high enough even in the new equilibrium.

We can show that Cutoff 1 < Cutoff 3, i.e.,

$$\frac{16I + (1-\mathcal{H})[6A - 2c_{NB}]}{(9+7\mathcal{H})} < \frac{(21-5\mathcal{H})I + (1-\mathcal{H})\left[2(9-2\mathcal{H})A - c_{NB}(9+\mathcal{H})\right]}{(9+7\mathcal{H})(1-\delta)}.$$

Therefore, if the economy starts with both bank dealers operating both services, increasing c_B and transitioning to the new equilibrium (above Cutoff 2) may temporarily reverse the result that overall customer welfare is increasing in c_B. However, this reversal is transitory, and when c_B is high enough (above Cutoff 3), we regain our previous result that overall customer welfare is increasing in c_B.

A Proofs

A.1 Proof of Proposition 1

The bank dealer's problem is

$$\pi_B(c_B) = \max_{0\leq f\leq S\leq c_{NB}} \Pi_B(c_B)$$

where

$$\Pi_B(c_B) = \frac{2\mu}{r}\left[(\mathcal{H}f - I)\left(G\left(\frac{S-\mathcal{H}f}{1-\mathcal{H}}\right) - G(f)\right) + (S - c_B)\left(1 - G\left(\frac{S-\mathcal{H}f}{1-\mathcal{H}}\right)\right)\right].$$

Consider the following change of variables

$$b = \frac{S-\mathcal{H}f}{1-\mathcal{H}},$$

then

$$\Pi_B(c_B) = \frac{2\mu}{r}\left[(\mathcal{H}f - I)(G(b) - G(f)) + ((1-\mathcal{H})b + \mathcal{H}f - c_B)(1 - G(b))\right], \tag{4}$$

the domain $\{(f,S)\,|0\leq f\leq S\leq c_{NB}\}$ becomes $\{(f,b)\,|0\leq f\leq b; (1-\mathcal{H})\,b + \mathcal{H}f \leq c_{NB}\}$, which is a triangle in the (f,b) space.

The first-order derivatives of (4) are

$$\frac{\partial \Pi_B}{\partial f} = \frac{2\mu}{r}\mathcal{H}G'(f)\left[\frac{1-G(f)}{G'(f)} - f + \frac{I}{\mathcal{H}}\right],$$

$$\frac{\partial \Pi_B}{\partial b} = \frac{2\mu}{r}(1-\mathcal{H})\,G'(b)\left(\frac{1-G(b)}{G'(b)} - b + \frac{c_B - I}{1-\mathcal{H}}\right).$$

Since $\phi(x) = x - \frac{1-G(x)}{G'(x)}$ is an increasing function, $\Pi_B(f,b)$ is an unimodal function of f with maximum satisfying $\phi(f_{max}) = \frac{I}{\mathcal{H}}$, and $\Pi_B(f,b)$ is an unimodal function of b with maximum satisfying $\phi(b_{max}) = \frac{c_B - I}{1-\mathcal{H}}$. One immediate observation is that

$$f_{max} < b_{max} \iff \frac{I}{\mathcal{H}} < \frac{c_B - I}{1-\mathcal{H}} \iff c_B > \frac{I}{\mathcal{H}}.$$

When $\phi(c_{NB}) \leq 0$, we must have $\phi(c_{NB}) < \frac{I}{\mathcal{H}}$, which implies $f_{max} > c_{NB}$. Then, the solution (f^*, b^*) either satisfies $f^* = b^*$, so there is no matchmaking, or satisfies the constraint $(1-\mathcal{H})\,b^* + \mathcal{H}f^* = c_{NB}$. When $c_B > \frac{I}{\mathcal{H}}$, suppose the bank dealer chooses $f = b = S$. It is straightforward to obtain

$$\frac{\partial \Pi_B}{\partial(-f)}|_{f=b=S} = \frac{2\mu}{r}\frac{\mathcal{H}}{1-\mathcal{H}}G'(f)\left(c_B - \frac{I}{\mathcal{H}}\right) > 0.$$

In this case, the bank dealer always has an incentive to lower the fee and therefore matchmaking must exist. The equilibrium must satisfy $(1-\mathcal{H})\,b^* + \mathcal{H}f^* = c_{NB}$, which is equivalent to $S^* = c_{NB}$. The equilibrium fee f^* is the minimal solution (by our equilibrium refinement) of

$$f^* = \arg\max_f \frac{2\mu}{r}\left[(\mathcal{H}f - I)\left(G\left(\frac{c_{NB}-\mathcal{H}f}{1-\mathcal{H}}\right) - G(f)\right) + (c_{NB} - c_B)\left(1 - G\left(\frac{c_{NB}-\mathcal{H}f}{1-\mathcal{H}}\right)\right)\right].$$

This is the constrained bank dealer equilibrium.

When $\phi(c_{NB}) > 0$, if $\frac{I}{\mathcal{H}} < \phi(c_{NB})$, we have $f_{max} < c_{NB}$. When $c_B > \frac{I}{\mathcal{H}}$, the optimal solution (f^*, b^*) is either (f_{max}, b_{max}) or it satisfies the constraint $(1-\mathcal{H})\,b^* + \mathcal{H}f^* = c_{NB}$. Define $\underline{c}$ as the solution of

$$\frac{1 - G\left(\frac{c_{NB}-\mathcal{H}f_{max}}{1-\mathcal{H}}\right)}{G'\left(\frac{c_{NB}-\mathcal{H}f_{max}}{1-\mathcal{H}}\right)} - \frac{c_{NB}-\mathcal{H}f_{max}}{1-\mathcal{H}} + \frac{\underline{c} - I}{1-\mathcal{H}} = 0.$$

It is easy to show that $\underline{c} \in \left(\frac{I}{\mathcal{H}}, c_{NB}\right)$. When $c_B < \underline{c}$, the constraint $(1-\mathcal{H})\,b + \mathcal{H}f \leq c_{NB}$ is not binding, which means that $(f^* = f_{max}, b^* = b_{max})$ must be the equilibrium. This is the unconstrained bank dealer equilibrium.

When $c_B \geq \underline{c}$, the constraint $(1-\mathcal{H})\, b + \mathcal{H} f \leq c_{NB}$ is binding, which implies $S^* = c_{NB}$ and $f^\star$ is the minimal solution (by our equilibrium refinement) of

$$f^* = \arg\max_f \frac{2\mu}{r} \left[(\mathcal{H} f - I) \left(G\left(\frac{c_{NB} - \mathcal{H} f}{1-\mathcal{H}} \right) - G(f) \right) + (c_{NB} - c_B) \left(1 - G\left(\frac{c_{NB} - \mathcal{H} f}{1-\mathcal{H}} \right) \right) \right].$$

This is also the constrained bank dealer equilibrium.

If $\frac{I}{\mathcal{H}} \geq \phi(c_{NB})$, we have $f_{max} \geq c_{NB}$. Then, the solution (f^*, b^*) either satisfies $f^* = b^*$, so there is no matchmaking, or satisfies the constraint $(1-\mathcal{H})\, b^* + \mathcal{H} f^* = c_{NB}$. When $c_B > \frac{I}{\mathcal{H}}$, suppose the bank dealer chooses $f = b = S$. In this case, we have

$$\frac{\partial \Pi_B}{\partial(-f)}|_{f=b=S} = \frac{2\mu}{r} \frac{\mathcal{H}}{1-\mathcal{H}} G'(f) \left(c_B - \frac{I}{\mathcal{H}} \right) > 0,$$

the bank dealer has an incentive to lower the fee, and therefore matchmaking must exist and the equilibrium must be constrained, $S^* = c_{NB}$. The equilibrium fee f^* is the minimal solution (by our equilibrium refinement) of

$$f^* = \arg\max_f \frac{2\mu}{r} \left[(\mathcal{H} f - I) \left(G\left(\frac{c_{NB} - \mathcal{H} f}{1-\mathcal{H}} \right) - G(f) \right) + (c_{NB} - c_B) \left(1 - G\left(\frac{c_{NB} - \mathcal{H} f}{1-\mathcal{H}} \right) \right) \right].$$

This is the constrained bank dealer equilibrium.

From this proposition, we know that when $c_B \in \left(\frac{I}{\mathcal{H}}, c_{NB} \right]$, the bank dealer always operates the matchmaking business. This implies that, if one starts with an equilibrium in which the parameters satisfy $c_B \in \left(\frac{I}{\mathcal{H}}, c_{NB} \right)$, then increasing the balance sheet cost locally or decreasing the matchmaking cost locally will still make the condition $c_B \in \left(\frac{I}{\mathcal{H}}, c_{NB} \right]$ hold, which means that matchmaking will exist in the new equilibrium as well. If one starts with an equilibrium in which the parameters satisfy $c_B = c_{NB}$ and $\frac{I}{\mathcal{H}} < c_{NB}$, then decreasing the matchmaking cost locally will still make the condition $c_B \in \left(\frac{I}{\mathcal{H}}, c_{NB} \right]$ hold, and in the proof of Proposition 2, we will show that increasing the balance sheet cost locally in this case will also lead to equilibrium with matchmaking.

A.2 Proof of Proposition 2

The non-bank dealer's profit function is

$$\Pi_{NB}(S) = \frac{2\mu}{r} \left[(S - c_{NB}) (1 - G(b)) \right].$$

To establish the existence of an equilibrium and find the equilibrium, we first conjecture that the equilibrium exists, and then verify it.

Suppose the equilibrium is (f^*, S^*). The first observation is that $f < \frac{I}{\mathcal{H}}$ is strictly dominated by $f = \frac{I}{\mathcal{H}}$. Then, given any $f \geq \frac{I}{\mathcal{H}}$, let us consider the properties of the non-bank dealer's best response function without considering the constraint $S \leq c_B$.

The first-order derivative of the non-bank dealer's profit is

$$\begin{aligned} \frac{d\pi_{NB}}{dS} &= \frac{2\mu}{r} \left[1 - G(b) - (S - c_{NB})\, G'(b) \frac{1}{1-\mathcal{H}} \right] \\ &= \frac{2\pi}{r} G'(b) \left[\frac{1 - G(b)}{G'(b)} - \frac{S - c_{NB}}{1-\mathcal{H}} \right]. \end{aligned}$$

We first verify that the matchmaking service must be offered in equilibrium. To see this, if there is no matchmaking in equilibrium, then the non-bank dealer's best response is $S_{BR} = \tilde{c}_B$. Since in this case $\frac{I}{\mathcal{H}} < \tilde{c}_B$, the bank dealer can always choose $f = \tilde{c}_B - \epsilon$ to obtain positive revenues. Thus, in equilibrium (if it exists), matchmaking services must be offered.

Given any $f \geq \frac{I}{\mathcal{H}}$, we can show

$$\frac{1 - G(b)}{G'(b)} - \frac{S - c_{NB}}{1-\mathcal{H}}|_{S=c_{NB}} > 0$$

$$\frac{1 - G(b)}{G'(b)} - \frac{S - c_{NB}}{1-\mathcal{H}}|_{S=\infty} < 0.$$

It is easy to see that $\frac{1-G(b)}{G'(b)} - \frac{S-c_{NB}}{1-\mathcal{H}}$ is strictly decreasing in S. Define $M(S,f) = \frac{1-G\left(\frac{S-\mathcal{H}f}{1-\mathcal{H}}\right)}{G'\left(\frac{S-\mathcal{H}f}{1-\mathcal{H}}\right)} - \frac{S-c_{NB}}{1-\mathcal{H}}$. Then, the best response $S_{BR}(f)$ is unique and is solved by

$$M(S_{BR}, f) = 0.$$

We want to find a uniform upper bound for the non-bank dealer's equilibrium strategy to help us use the Kakutani Fixed-Point Theorem later. The best response is solved by

$$\begin{aligned} M(S_{BR}, f) = 0 \Longleftrightarrow \frac{S_{BR}(f) - c_{NB}}{1-\mathcal{H}} &= \frac{1 - G\left(\frac{S_{BR}(f)-\mathcal{H}f}{1-\mathcal{H}}\right)}{G'\left(\frac{S_{BR}(f)-\mathcal{H}f}{1-\mathcal{H}}\right)} \leq \frac{1-G(0)}{G'(0)} \\ \Longleftrightarrow S_{BR}(f) &\leq c_{NB} + (1-\mathcal{H})\frac{1-G(0)}{G'(0)}. \end{aligned}$$

So WLOG, we just focus on the non-bank dealer's strategy space $S \in \left[c_{NB}, c_{NB} + (1-\mathcal{H})\frac{1-G(0)}{G'(0)}\right]$.

Since M is continuously differentiable in both S and f, we know that the unique best response function $S_{BR}(f)$ is also continuous. Besides,

$$\frac{\partial M}{\partial S} dS_{BR} + \frac{\partial M}{\partial f} df = 0 \Longrightarrow \frac{dS_{BR}}{df} = -\frac{\frac{\partial M}{\partial f}}{\frac{\partial M}{\partial S}}.$$

It is clear that

$$\frac{\partial M}{\partial f} > 0, \frac{\partial M}{\partial S} < 0,$$

so we must have

$$\frac{dS_{BR}}{df} > 0.$$

Hence, $S_{BR}(f)$ is strictly increasing in f.

Consider the bank dealer's best response function. Given the non-bank dealer's choice

$$S \in \left[c_{NB}, c_{NB} + (1-\mathcal{H})\frac{1-G(0)}{G'(0)}\right],$$

the first-order derivative of the bank dealer's profit function is

$$\begin{aligned} \frac{\partial \pi_B}{\partial f} &= \mathcal{H}(G(b) - G(f)) + (\mathcal{H}f - I)\left(-\frac{\mathcal{H}}{1-\mathcal{H}}G'(b) - G'(f)\right) \\ &= \mathcal{H}G'(f)\left[\frac{1-G(f)}{G'(f)} - f + \frac{I}{\mathcal{H}}\right] - \mathcal{H}G'(b)\left(\frac{\mathcal{H}f - I}{1-\mathcal{H}} + \frac{1-G(b)}{G'(b)}\right) \\ &= \mathcal{H}G'(f)\left[\frac{1-G(f)}{G'(f)} - \frac{\mathcal{H}f - I}{\mathcal{H}}\right] - \mathcal{H}G'(b)\left(\frac{1-G(b)}{G'(b)} + \frac{\mathcal{H}f - I}{1-\mathcal{H}}\right). \end{aligned}$$

It it clear that $\frac{\partial \pi_B}{\partial f}|_{f=\frac{I}{\mathcal{H}}} > 0$ and $\frac{\partial \pi_B}{\partial f}|_{f=S} < 0$, so the best response $f_{BR}(S)$ must be interior. Thus, the FOC must be satisfied and we have

$$\frac{1-G(f_{BR})}{G'(f_{BR})} - f_{BR} + \frac{I}{\mathcal{H}} > 0 \Longleftrightarrow \phi(f_{BR}) < \frac{I}{\mathcal{H}}.$$

The best response must satisfy $f_{BR} \in \left[\frac{I}{\mathcal{H}}, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right]$. In region $f \in \left[\frac{I}{\mathcal{H}}, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right]$,

1. if G is concave, then $G'(f)$ is decreasing in f . Besides, $\frac{1-G(f)}{G'(f)} - f + \frac{I}{\mathcal{H}}$ is positive and decreasing in f, $G'(b)$ is increasing in f, $\frac{\mathcal{H}f-I}{1-\mathcal{H}} + \frac{1-G(b)}{G'(b)}$ is increasing in f, and therefore $\frac{\partial \pi_B}{\partial f}$ is decreasing in $f \in \left[\frac{I}{\mathcal{H}}, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right]$. This implies that the best response must be unique, and we can show that the best response function is continuous by the implicit function theorem.

2. if G is convex, $G''' \leq 0$ and $\mathcal{H} \leq \frac{1}{2}$, we can rewrite $\frac{\partial \pi_B}{\partial f}$ as

$$\frac{\partial \pi_B}{\partial f} = \mathcal{H}(G(b) - G(f)) - \frac{(\mathcal{H}f - I)}{1-\mathcal{H}}(\mathcal{H}G'(b) + (1-\mathcal{H})G'(f)).$$

When $f \geq \frac{I}{\mathcal{H}}$, we know $\mathcal{H}(G(b) - G(f))$ is decreasing in f, and $\frac{(\mathcal{H}f - I)}{1-\mathcal{H}}$ is positive and increasing in f. If we can show that $(\mathcal{H}G'(b) + (1-\mathcal{H})G'(f))$ is increasing in f, then $\frac{\partial \pi_B}{\partial f}$ would be monotonically decreasing in f when $f \geq \frac{I}{\mathcal{H}}$, implying that the best response is unique. The following claim confirms this result.

Claim 1. If $\mathcal{H} < \frac{1}{2}$ and $G'''(x) < 0$ for all x, then $\left(\mathcal{H}G'\left(\frac{c_B - \mathcal{H}f}{1-\mathcal{H}}\right) + (1-\mathcal{H})G'(f)\right)$ is increasing in f.

Proof. $\mathcal{H}b + (1-\mathcal{H})f = f + \mathcal{H}(b-f) = f + \frac{\mathcal{H}}{1-\mathcal{H}}(c_B - f) = \frac{\mathcal{H}}{1-\mathcal{H}}c_B + \frac{1-2\mathcal{H}}{1-\mathcal{H}}f$. Consider any f_1 and f_2 that satisfy $\frac{I}{\mathcal{H}} \leq f_1 < f_2 \leq c_B$. Define $\delta f = f_2 - f_1 > 0$, $b_1 = \frac{c_B - \mathcal{H}f_1}{1-\mathcal{H}}$, $b_2 = \frac{c_B - \mathcal{H}f_2}{1-\mathcal{H}}$, and $\delta b = b_2 - b_1 = -\frac{\mathcal{H}}{1-\mathcal{H}}\delta f < 0$. Then,

$$(\mathcal{H}G'(b_2) + (1-\mathcal{H})G'(f_2)) \geq \left(\mathcal{H}G'\left(b_2 - \frac{1-2\mathcal{H}}{1-\mathcal{H}}\delta f\right) + (1-\mathcal{H})G'\left(f_2 - \frac{1-2\mathcal{H}}{1-\mathcal{H}}\delta f\right)\right)$$

because $G'' > 0$. And

$$\left(\mathcal{H}G'\left(b_2 - \frac{1-2\mathcal{H}}{1-\mathcal{H}}\delta f\right) + (1-\mathcal{H})G'\left(f_2 - \frac{1-2\mathcal{H}}{1-\mathcal{H}}\delta f\right)\right) \geq (\mathcal{H}G'(b_1) + (1-\mathcal{H})G'(f_1))$$

because G' is concave, and

$$\mathcal{H}\left(b_2 - \frac{1-2\mathcal{H}}{1-\mathcal{H}}\delta f\right) + (1-\mathcal{H})\left(f_2 - \frac{1-2\mathcal{H}}{1-\mathcal{H}}\delta f\right) = \mathcal{H}b_1 + (1-\mathcal{H})f_1.$$

□

Since f_{BR} is continuous on $\left[\frac{I}{\mathcal{H}}, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right]$and S_{BR} is continuous on $\left[c_{NB}, c_{NB} + (1-\mathcal{H})\frac{1-G(0)}{G'(0)}\right]$, by the Kakutani Fixed Point Theorem the equilibrium must exist, and it satisfies

$$\frac{1-G(b)}{G'(b)} - \frac{S - c_{NB}}{1-\mathcal{H}} = 0, \tag{5}$$

$$\mathcal{H}G'(f)\left[\frac{1-G(f)}{G'(f)} - \frac{\mathcal{H}f - I}{\mathcal{H}}\right] - \mathcal{H}G'(b)\left(\frac{1-G(b)}{G'(b)} + \frac{\mathcal{H}f - I}{1-\mathcal{H}}\right) = 0. \tag{6}$$

We are now going to show that there is a unique solution satisfying (5) and (6). Suppose there exist two solutions of the above two conditions: (f_1, S_1) and (f_2, S_2). WLOG we can assume $f_1 < f_2$. Then, we must have $S_1 < S_2$ because $S_{BR}(f)$ is strictly increasing in f. Using (5) and $S_1 < S_2$, we can show that $b_1 > b_2$.

1. If G is concave, since (f_1, S_1) satisfies (6), we know

$$\mathcal{H}G'(f_1)\left[\frac{1-G(f_1)}{G'(f_1)} - \frac{\mathcal{H}f_1 - I}{\mathcal{H}}\right] = \mathcal{H}G'(b_1)\left(\frac{1-G(b_1)}{G'(b_1)} + \frac{\mathcal{H}f_1 - I}{1-\mathcal{H}}\right). \tag{7}$$

Since $f_1 < f_2$, and $\frac{1-G(f_i)}{G'(f_i)} - \frac{\mathcal{H}f_i - I}{\mathcal{H}} > 0$ is satisfied for both $i = 1, 2$, we have

$$\frac{1-G(f_1)}{G'(f_1)} - \frac{\mathcal{H}f_1 - I}{\mathcal{H}} > \frac{1-G(f_2)}{G'(f_2)} - \frac{\mathcal{H}f_2 - I}{\mathcal{H}} > 0.$$

Besides, we have $G'(f_1) > G'(f_2)$. So

$$\mathcal{H}G'(f_1)\left[\frac{1-G(f_1)}{G'(f_1)} - \frac{\mathcal{H}f_1 - I}{\mathcal{H}}\right] > \mathcal{H}G'(f_2)\left[\frac{1-G(f_2)}{G'(f_2)} - \frac{\mathcal{H}f_2 - I}{\mathcal{H}}\right]. \tag{8}$$

Similarly, since $b_1 > b_2$ and $f_1 < f_2$, we have

$$\mathcal{H}G'(b_1)\left(\frac{1-G(b_1)}{G'(b_1)} + \frac{\mathcal{H}f_1 - I}{1-\mathcal{H}}\right) < \mathcal{H}G'(b_2)\left(\frac{1-G(b_2)}{G'(b_2)} + \frac{\mathcal{H}f_2 - I}{1-\mathcal{H}}\right). \tag{9}$$

Equations (7), (8), and (9) imply that

$$\mathcal{H}G'(f_2)\left[\frac{1-G(f_2)}{G'(f_2)} - \frac{\mathcal{H}f_2 - I}{\mathcal{H}}\right] < \mathcal{H}G'(b_2)\left(\frac{1-G(b_2)}{G'(b_2)} + \frac{\mathcal{H}f_2 - I}{1-\mathcal{H}}\right).$$

But this is impossible because by the definition of (f_2, S_2), we know

$$\mathcal{H}G'(f_2)\left[\frac{1-G(f_2)}{G'(f_2)} - \frac{\mathcal{H}f_2 - I}{\mathcal{H}}\right] = \mathcal{H}G'(b_2)\left(\frac{1-G(b_2)}{G'(b_2)} + \frac{\mathcal{H}f_2 - I}{1-\mathcal{H}}\right).$$

Hence, there must be a unique solution satisfying (5) and (6).

2. If G is convex, $\mathcal{H} < \frac{1}{2}$ and $G''' < 0$, we know that

$$\mathcal{H}(G(b_1) - G(f_1)) > \mathcal{H}(G(b_2) - G(f_2)).$$

By Claim 1, we know that

$$\begin{aligned}\left(\mathcal{H}G'\left(\frac{S_1 - \mathcal{H}f_1}{1-\mathcal{H}}\right) + (1-\mathcal{H})G'(f_1)\right) &< \left(\mathcal{H}G'\left(\frac{S_1 - \mathcal{H}f_2}{1-\mathcal{H}}\right) + (1-\mathcal{H})G'(f_2)\right) \\ &< \left(\mathcal{H}G'\left(\frac{S_2 - \mathcal{H}f_2}{1-\mathcal{H}}\right) + (1-\mathcal{H})G'(f_2)\right).\end{aligned}$$

Since

$$\mathcal{H}(G(b_1) - G(f_1)) - (\mathcal{H}G'(b_1) + (1-\mathcal{H})G'(f_1)) = 0,$$

we must have

$$\mathcal{H}(G(b_2) - G(f_2)) - (\mathcal{H}G'(b_2) + (1-\mathcal{H})G'(f_2)) < 0,$$

which violates the FOC. Hence, there must be a unique solution satisfying (5) and (6).

Denote the equilibrium as $(\bar{f}^*, \bar{S}^*)$, and define $\bar{c} = \bar{S}^*$. When $c_B > \bar{c}$, the unique equilibrium is $(\bar{f}^*, \bar{S}^*)$, and this is the unconstrained equilibrium. When $c_B \leq \bar{c}$, in equilibrium we must have $S^* = c_B$ (because otherwise it will be the unconstrained equilibrium). The bank dealer's unique best response f^* is solved by

$$\mathcal{H}G'(f^*)\left[\frac{1-G(f^*)}{G'(f^*)} - \frac{\mathcal{H}f^* - I}{\mathcal{H}}\right] - \mathcal{H}G'\left(\frac{c_B - \mathcal{H}f^*}{1-\mathcal{H}}\right)\left(\frac{1-G\left(\frac{c_B-\mathcal{H}f^*}{1-\mathcal{H}}\right)}{G'\left(\frac{c_B-\mathcal{H}f^*}{1-\mathcal{H}}\right)} + \frac{\mathcal{H}f^* - I}{1-\mathcal{H}}\right) = 0.$$

Because the non-bank dealer's best response is increasing in f, $S^* = c_B$ must be her best response, which implies that (f^*, c_B) is the unique equilibrium.

From this proposition, we know that when $c_B > c_{NB}$ and $I < \mathcal{H}\min\{\tilde{c}_B, c_B\}$, the bank dealer always operates the matchmaking business. This implies that, if one starts with an equilibrium in which the parameters satisfy $c_B > c_{NB}$ and $I < \mathcal{H}\min\{\tilde{c}_B, c_B\}$, then increasing the balance sheet cost locally or decreasing the matchmaking cost locally will still make the condition $c_B > c_{NB}$ and $I < \mathcal{H}\min\{\tilde{c}_B, c_B\}$ hold, which means that matchmaking will still exist in the new equilibrium. If one starts with an equilibrium in which the parameters satisfying $c_B = c_{NB}$ and $\frac{I}{\mathcal{H}} < c_{NB}$, from Proposition 1 we know that matchmaking exists in this equilibrium. Increasing the balance sheet cost locally in this case will make the condition $c_B > c_{NB}$ and $I < \mathcal{H}\min\{\tilde{c}_B, c_B\}$ hold, thus the matchmaking service will still exist in the new equilibrium.

A.3 Proof of Proposition 3

In this equilibrium, obviously $S^* = c_{NB}$ is unchanged. f^* is the minimal solution of the following problem

$$f^* = \arg\max_f \frac{2\mu}{r}\left[(\mathcal{H}f - I)\left(G\left(\frac{c_{NB} - \mathcal{H}f}{1-\mathcal{H}}\right) - G(f)\right) + (c_{NB} - c_B)\left(1 - G\left(\frac{c_{NB} - \mathcal{H}f}{1-\mathcal{H}}\right)\right)\right].$$

Since

$$\frac{\partial^2\left[(\mathcal{H}f - I)\left(G\left(\frac{c_{NB}-\mathcal{H}f}{1-\mathcal{H}}\right) - G(f)\right) + (c_{NB} - c_B)\left(1 - G\left(\frac{c_{NB}-\mathcal{H}f}{1-\mathcal{H}}\right)\right)\right]}{\partial c_B \partial f} < 0,$$

by Topkis's theorem, when c_B increases, f^* must decrease. Therefore, $b^* = \frac{c_{NB}-\mathcal{H}f^*}{1-\mathcal{H}}$ must increase.

Average transaction costs is

$$\begin{aligned}ATC &= \frac{(G(b^*) - G(f^*))f^* + (1 - G(b^*))S^*}{1 - G(f^*)} \\ &= S^* - \frac{G(b^*) - G(f^*)}{1 - G(f^*)}(S^* - f^*). \qquad (10)\end{aligned}$$

When c_B increases, we know f^* decreases and b^* increases, thus

$$\frac{G(b^*) - G(f^*)}{1 - G(f^*)} = \frac{G(b^*) - G(f^*)}{1 - G(b^*) + G(b^*) - G(f^*)} = \frac{1}{1 + \frac{1-G(b^*)}{G(b^*)-G(f^*)}}$$

increases. We also know that $(S^* - f^*)$ increases, which means that ATC decreases.

Trading volume $(1 - G(f^*))$ must increase because f^* decreases. Matchmaking $(G(b^*) - G(f^*))$ must increase because b^* increases, which also implies that market making $(1 - G(b^*))$ decreases.

Overall customer welfare is

$$\pi_c = \int_{f^*}^{b^*} \mathcal{H}(x - f^*) \, dG(x) + \int_{b^*}^{\infty} (x - S^*) \, dG(x),$$

so

$$\frac{d\pi_c}{dc_B} = \int_{f^*}^{b^*} \mathcal{H} \frac{-df^*}{dc_B} dG(x) > 0.$$

A.4 Proof of Proposition 4

The equilibrium fee f^* can be found from $\phi(f^*) = \frac{I}{\mathcal{H}}$, so it is independent of c_B. b^* is the solution to $\phi(b^*) = \frac{c_B - I}{1-\mathcal{H}}$, so b^* is increasing in c_B. Thus, $S^* = (1-\mathcal{H}) b^* + \mathcal{H} f^*$ is increasing in c_B. Average transaction costs is

$$\begin{aligned} ATC &= \frac{(G(b^*) - G(f^*)) f^* + (1 - G(b^*)) S^*}{1 - G(f^\star)} \\ &= \frac{(G(b^*) - G(f^*)) f^* + (1 - G(b^*)) (\mathcal{H} f^* + (1-\mathcal{H}) b^*)}{1 - G(f^*)}. \end{aligned}$$

Then,

$$\begin{aligned} \frac{dATC}{dc_B} &= \frac{db^*}{dc_B} \cdot \frac{G'(b^*) f^* + (-G'(b^*))(\mathcal{H} f^* + (1-\mathcal{H}) b^*) + (1 - G(b^*))(1-\mathcal{H})}{1 - G(f^*)} \\ &= \frac{db^*}{dc_B} \cdot \frac{G'(b^*)(1-\mathcal{H})}{1 - G(f^*)} \left(f^\star - b^* + \frac{1 - G(b^*)}{G'(b^*)} \right) \\ &= \frac{db^*}{dc_B} \cdot \frac{G'(b^*)(1-\mathcal{H})}{1 - G(f^*)} \left(f^* - \frac{c_B - I}{1 - \mathcal{H}} \right). \end{aligned}$$

When $c_B < (1-\mathcal{H}) f^* + I$, average transaction costs are increasing in c_B; when $c_B \geq (1-\mathcal{H}) f^* + I$, average transaction costs are decreasing in c_B.

Since f^* is unchanged and b^* increases, trading volume $(1 - G(f^*))$ must be unchanged. Matchmaking $(G(b^*) - G(f^*))$ increases, and market making $(1 - G(b^*))$ decreases.

Overall customer welfare is

$$\pi_c = \frac{2\mu}{r} \left[\int_{f^*}^{b^*} \mathcal{H}(x - f^*) \, dG(x) + \int_{b^*}^{\infty} (x - S^*) \, dG(x) \right],$$

thus

$$\frac{d\pi_c}{dc_B} = \frac{2\mu}{r} \int_{b^*}^{\infty} -\frac{dS^*}{dc_B} dG(x) < 0.$$

A.5 Proof of Proposition 5

When establishing the existence of an equilibrium in this case, we already imposed the following additional assumptions on G:

- $G(x)$ is a concave function, or
- $G(x)$ is a convex function, and $\mathcal{H} < \frac{1}{2}$, $G'''(x) < 0$ for all x.

In the constrained non-bank dealer equilibrium, the bank dealer's profit is

$$\Pi_B = \frac{2\mu}{r}\left(\mathcal{H}f - I\right)\left(G\left(\frac{c_B - \mathcal{H}f}{1-\mathcal{H}}\right) - G\left(f\right)\right).$$

In equilibrium, we have

$$\frac{\partial^2 \pi_B}{\partial f \partial c_B} = \frac{2\mu}{r}\frac{\mathcal{H}}{1-\mathcal{H}}\left(G'\left(b^*\right) - \frac{\mathcal{H}f^* - I}{1-\mathcal{H}}G''\left(b^*\right)\right).$$

When G is concave, it is clear that $\frac{\partial^2 \pi_B}{\partial f \partial S} > 0$. When G is convex, in equilibrium the FOC is satisfied:

$$\mathcal{H}\left(G\left(b^*\right) - G\left(f^*\right)\right) + \left(\mathcal{H}f^* - I\right)\left(-\frac{\mathcal{H}}{1-\mathcal{H}}G'\left(b^*\right) - G'\left(f^*\right)\right) = 0$$
$$\Longleftrightarrow G\left(b^*\right) - G\left(f^*\right) = \frac{\mathcal{H}f^* - I}{\mathcal{H}\left(1-\mathcal{H}\right)}\left(\mathcal{H}G'\left(b^*\right) + \left(1-\mathcal{H}\right)G'\left(f^*\right)\right).$$

Since G is convex, we have $G\left(b^*\right) - G\left(f^*\right) \leq \left(b^* - f^*\right)G'\left(b^*\right)$, thus

$$\frac{\mathcal{H}f^* - I}{\mathcal{H}\left(1-\mathcal{H}\right)}\left(\mathcal{H}G'\left(b^*\right) + \left(1-\mathcal{H}\right)G'\left(f^*\right)\right) \leq \left(b^* - f^*\right)G'\left(b^*\right)$$
$$\Longleftrightarrow \left(f^* - \frac{I}{\mathcal{H}}\right)\left(\mathcal{H}G'\left(b^*\right) + \left(1-\mathcal{H}\right)G'\left(f^*\right)\right) \leq \left(c_B - f^*\right)G'\left(b^*\right)$$
$$\Longleftrightarrow \left(f^* - \frac{I}{\mathcal{H}}\right)\left(1-\mathcal{H}\right)G'\left(f^*\right) \leq \left(c_B - f^* - \left(\mathcal{H}f^* - I\right)\right)G'\left(b^*\right).$$

This implies that $c_B - f^* - \left(\mathcal{H}f^* - I\right) > 0$. Moreover, when G' is concave, we have $G''\left(b^*\right) \leq \frac{G'\left(b^*\right) - G'\left(f^*\right)}{b^* - f^*} < \frac{G'\left(b^*\right)}{b^* - f^*}$. Then,

$$\begin{aligned} G'\left(b^*\right) - \frac{\mathcal{H}f^* - I}{1-\mathcal{H}}G''\left(b^*\right) &> G'\left(b^*\right) - \frac{\mathcal{H}f^* - I}{1-\mathcal{H}}\frac{G'\left(b^*\right)}{b^* - f^*} \\ &= G'\left(b^*\right)\frac{c_B - f^* - \left(\mathcal{H}f^* - I\right)}{c_B - f^*} \\ &> 0. \end{aligned}$$

Hence, in both cases we have $\frac{\partial^2 \pi_B}{\partial f \partial S}|_{f^*} > 0$, and by Topkis's theorem f^* must be increasing in c_B. To consider the change in b^*,

$$\begin{aligned} \frac{\partial \Pi_B}{\partial f} &= \frac{2\mu}{r}\left[\mathcal{H}\left(G\left(b^*\right) - G\left(f^*\right)\right) + \left(\mathcal{H}f^* - I\right)\left(-\frac{\mathcal{H}}{1-\mathcal{H}}G'\left(b^*\right) - G'\left(f^*\right)\right)\right] \\ &= \frac{2\mu}{r}\left[\mathcal{H}G'\left(f\right)\left(\frac{1-G\left(f\right)}{G'\left(f\right)} - f + \frac{I}{\mathcal{H}}\right) - \mathcal{H}\left(1-G\left(b\right)\right) - \left(\mathcal{H}f - I\right)\frac{\mathcal{H}}{1-\mathcal{H}}G'\left(b\right)\right]. \end{aligned}$$

The optimal solution f^* must satisfy $f^* \in \left(\frac{I}{\mathcal{H}}, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right)$. In the proof of the equilibrium existence, we already showed that $\Pi_B\left(f\right)$ is unimodal in $f^* \in \left(\frac{I}{\mathcal{H}}, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right)$ with the additional assumptions on G. This implies that if $\frac{\partial \Pi_B}{\partial f}\left(x; c_B\right) < 0$, then $\frac{\partial \Pi_B}{\partial f}\left(f; c_B\right) < 0$ for all $f \in \left(x, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right)$. Suppose when $c_B = c_{B1}$, the equilibrium is $f^* = f_1$. When c_B increases to c_{B2}, denote the equilibrium as f_2. Suppose f increases to $\tilde{f}_2$ in this case such that

$$b_1 = \frac{c_{B1} - \mathcal{H}f_1}{1-\mathcal{H}} = \frac{c_{B2} - \mathcal{H}\tilde{f}_2}{1-\mathcal{H}}.$$

Then,

$$\frac{\partial \Pi_B}{\partial f}\left(\tilde{f}_2; c_{B2}\right) = \frac{2\mu}{r}\left[\mathcal{H}G'\left(\tilde{f}_2\right)\left(\frac{1-G\left(\tilde{f}_2\right)}{G'\left(\tilde{f}_2\right)} - \tilde{f}_2 + \frac{I}{\mathcal{H}}\right) - \mathcal{H}\left(1-G\left(b_1\right)\right) - \left(\mathcal{H}\tilde{f}_2 - I\right)\frac{\mathcal{H}}{1-\mathcal{H}}G'\left(b_1\right)\right].$$

The optimality of f_1 implies

$$\frac{\partial \Pi_B}{\partial f}\left(f_1; c_{B1}\right) = \frac{2\mu}{r}\left[\mathcal{H}G'\left(f_1\right)\left(\frac{1-G\left(f_1\right)}{G'\left(f_1\right)} - f_1 + \frac{I}{\mathcal{H}}\right) - \mathcal{H}\left(1-G\left(b_1\right)\right) - \left(\mathcal{H}f_1 - I\right)\frac{\mathcal{H}}{1-\mathcal{H}}G'\left(b_1\right)\right] = 0.$$

If G is concave, it is straightforward to show that $\frac{\partial \Pi_B}{\partial f}\left(\tilde{f}_2; c_{B2}\right) < \frac{\partial \Pi_B}{\partial f}\left(f_1; c_{B1}\right) = 0$, so $\frac{\partial \Pi_B}{\partial f}\left(f; c_{B2}\right) < 0$ for all $f \in \left(\tilde{f}_2, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right)$, and we must have $f_2 < \tilde{f}_2$, which implies that $b_2 = \frac{c_{B2} - \mathcal{H} f_2}{1 - \mathcal{H}} > b_1$.

Similarly, if G is convex,

$$\frac{\partial \Pi_B}{\partial f}\left(\tilde{f}_2; c_{B2}\right) = \frac{2\mu}{r}\left[\mathcal{H}\left(1 - G\left(\tilde{f}_2\right) - \left(\tilde{f}_2 - \frac{I}{\mathcal{H}}\right) G'\left(\tilde{f}_2\right)\right) - \mathcal{H}\left(1 - G\left(b_1\right)\right) - \left(\mathcal{H}\tilde{f}_2 - I\right)\frac{\mathcal{H}}{1 - \mathcal{H}} G'\left(b_1\right)\right].$$

We know that

$$\frac{\partial \Pi_B}{\partial f}\left(f_1; c_{B1}\right) = \frac{2\mu}{r}\left[\mathcal{H}\left(1 - G\left(f_1\right) - \left(f_1 - \frac{I}{\mathcal{H}}\right) G'\left(f_1\right)\right) - \mathcal{H}\left(1 - G\left(b_1\right)\right) - \left(\mathcal{H} f_1 - I\right)\frac{\mathcal{H}}{1 - \mathcal{H}} G'\left(b_1\right)\right] = 0.$$

Besides, when f_1 increases (and we keep b_1 unchanged), $1 - G\left(f_1\right) - \left(f_1 - \frac{I}{\mathcal{H}}\right) G'\left(f_1\right)$ must be decreasing when $f_1 \geq \frac{I}{\mathcal{H}}$. Therefore, the RHS of the above expression is decreasing in f_1, which implies $\frac{\partial \Pi_B}{\partial f}\left(\tilde{f}_2; c_{B2}\right) < \frac{\partial \Pi_B}{\partial f}\left(f_1; c_{B1}\right) = 0$, so $\frac{\partial \Pi_B}{\partial f}\left(f; c_{B2}\right) < 0$ for all $f \in \left(\tilde{f}_2, \phi^{-1}\left(\frac{I}{\mathcal{H}}\right)\right)$. We therefore must have $f_2 < \tilde{f}_2$ and as a result also $b_2 = \frac{c_{B2} - \mathcal{H} f_2}{1 - \mathcal{H}} > b_1$. Hence, we can conclude that $b^\star$ is increasing in c_B.

From the above analysis we can show that trading volume $(1 - G(f^*))$ is decreasing in c_B and market making $(1 - G(b^*))$ is decreasing in c_B.

Overall customer welfare is

$$\pi_c = \int_{f^*}^{b^*} \mathcal{H}\left(x - f^\star\right) dG(x) + \int_{b^*}^{\infty} \left(x - S^*\right) dG(x).$$

Hence,

$$\frac{d\pi_c}{dc_B} = \int_{f^*}^{b^*} \mathcal{H}\left(-\frac{df^*}{dc_B}\right) dG(x) + \int_{b^*}^{\infty}\left(-\frac{dS^*}{dx}\right) dG(x) < 0.$$

A.6 Proof of Proposition 6

When I decreases, it is clear that S^* is unchanged in both the constrained bank dealer equilibrium and the constrained non-bank dealer equilibrium.

In the constrained bank dealer equilibrium, f^* is the minimal solution (by our equilibrium refinement) of

$$f^* = \arg\max_f \frac{2\mu}{r}\left[\left(\mathcal{H} f - I\right)\left(G\left(\frac{c_{NB} - \mathcal{H} f}{1 - \mathcal{H}}\right) - G(f)\right) + \left(c_{NB} - c_B\right)\left(1 - G\left(\frac{c_{NB} - \mathcal{H} f}{1 - \mathcal{H}}\right)\right)\right].$$

It is straightforward to show that

$$\frac{\partial^2\left[\left(\mathcal{H} f - I\right)\left(G\left(\frac{c_{NB} - \mathcal{H} f}{1 - \mathcal{H}}\right) - G(f)\right) + \left(c_{NB} - c_B\right)\left(1 - G\left(\frac{c_{NB} - \mathcal{H} f}{1 - \mathcal{H}}\right)\right)\right]}{\partial I \partial f} > 0,$$

and therefore

$$\frac{df^*}{dI} > 0.$$

Hence, when I decreases, f^* decreases as well.

In the constrained non-bank dealer equilibrium, f^* is solved by

$$f^* = \arg\max_f \left(\mathcal{H} f - I\right)\left(G\left(\frac{c_B - \mathcal{H} f}{1 - \mathcal{H}}\right) - G(f)\right).$$

It is clear that the objective function has an increasing difference in (I, f). Hence, when I decreases, f^* decreases as well.

Therefore, in both cases we have that b^* increases because f^* decreases and S^* is unchanged. Thus, $G(f^*)$ decreases, $G(b^*) - G(f^*)$ increases, and $1 - G(b^*)$ decreases.

Average transaction costs is

$$\begin{aligned} ATC &= \frac{\left(G(b^*) - G(f^*)\right) f^* + \left(1 - G(b^*)\right) S^*}{1 - G(f^*)} \\ &= S^\star - \frac{G(b^*) - G(f^*)}{1 - G(f^*)}\left(S^* - f^*\right). \end{aligned} \tag{11}$$

When I decreases, we know that f^* decreases and b^* increases, thus

$$\frac{G(b^*) - G(f^*)}{1 - G(f^*)} = \frac{G(b^*) - G(f^*)}{1 - G(b^*) + G(b^*) - G(f^*)} = \frac{1}{1 + \frac{1-G(b^*)}{G(b^*)-G(f^*)}}$$

increases. We also know that $(S^* - f^*)$ increases. Hence, ATC decreases.

Overall customer welfare is

$$\pi_c = \frac{2\mu}{r}\left[\int_{f^*}^{b^*} \mathcal{H}(x - f^*)\, dG(x) + \int_{b^*}^{\infty} (x - S^*)\, dG(x)\right],$$

We can obtain

$$\frac{d\pi_c}{dI} \propto \int_{f^*}^{b^*} \mathcal{H}\left(-\frac{df^*}{dI}\right) dG(x) < 0.$$

Hence, when I decreases, overall customer welfare increases.

A.7 Proof of Proposition 7

In the unconstrained bank dealer equilibrium, the equilibrium (S^*, f^*, b^*) is solved using the following conditions:

$$\frac{1 - G(f^*)}{G'(f^*)} - f^* + \frac{I}{\mathcal{H}} = 0,$$

$$\frac{1 - G(b^*)}{G'(b^*)} - b^* + \frac{c_B - I}{1 - \mathcal{H}} = 0;$$

$$S^* = \mathcal{H} f^* + (1 - \mathcal{H})\, b^*.$$

Recall that

$$\phi(x) = x - \frac{1 - G(x)}{G'(x)} = x - \zeta(x)$$

is increasing in x. From the above equilibrium conditions we have

$$\frac{1}{\mathcal{H}} = \phi'(f^*)\frac{df^*}{dI}, \tag{12}$$

$$\frac{1}{1 - \mathcal{H}} = \phi'(b^*)\left(-\frac{db^*}{dI}\right), \tag{13}$$

$$\frac{dS^*}{dI} = \mathcal{H}\frac{df^*}{dI} + (1 - \mathcal{H})\frac{db^*}{dI}.$$

Therefore, $\frac{df^*}{dI} > 0$ and $-\frac{db^*}{dI} > 0$. Then, when I decreases, f^* decreases and b^* increases. It is also straightforward to show that

$$\begin{aligned}\frac{dS^*}{dI} &= \mathcal{H}\frac{df^*}{dI} + (1 - \mathcal{H})\frac{db^*}{dI} \\ &= \frac{1}{\phi'(f^*)} - \frac{1}{\phi'(b^*)}.\end{aligned}$$

If $\phi'(\cdot)$ is an increasing function, i.e., ζ is concave, then $\frac{1}{\phi'(f^*)} - \frac{1}{\phi'(b^*)} \geq 0$, thus S^* decreases when I decreases. Similarly, when ζ is convex, we can show that S^* increases when I decreases.

From our results on f^* and b^*, we know that trading volume $(1 - G(f^*))$ increases, matchmaking $(G(b^*) - G(f^*))$ increases, and market making $(1 - G(b^*))$ decreases.

Overall customer welfare is

$$\pi_c = \frac{2\mu}{r}\left[\int_{f^*}^{b^*} \mathcal{H}(x - f^*)\, dG(x) + \int_{b^*}^{\infty} (x - S^*)\, dG(x)\right].$$

Then,

$$\begin{aligned}\frac{d\pi_c}{dI} &= \frac{2\mu}{r}\left[\int_{f^*}^{b^*}\mathcal{H}\left(-\frac{df^*}{dI}\right)dG(x) + \int_{b^*}^{\infty}\left(-\frac{dS^*}{dI}\right)dG(x)\right] \\ &= \frac{2\mu}{r}\left[-\mathcal{H}\frac{df^*}{dI}\left(G(b^\star) - G(f^*)\right) - (1-G(b^*))\frac{dS^\star}{dI}\right] \\ &= \frac{2\mu}{r}\left[-\mathcal{H}\frac{df^*}{dI}\left(G(b^*) - G(f^*)\right) - (1-G(b^*))\left(\mathcal{H}\frac{df^*}{dI} + (1-\mathcal{H})\frac{db^*}{dI}\right)\right] \\ &= \frac{2\mu}{r}\left[-\mathcal{H}\frac{df^*}{dI}\left(1 - G(f^*)\right) - (1-G(b^*))(1-\mathcal{H})\frac{db^*}{dI}\right]\end{aligned}$$

Thus,

$$\begin{aligned}\frac{d\pi_c}{dI} \leq 0 &\Longleftrightarrow -\mathcal{H}\frac{df^*}{dI}\left(1-G(f^*)\right) - (1-G(b^*))(1-\mathcal{H})\frac{db^*}{dI} \leq 0 \\ &\Longleftrightarrow (1-G(b^*))(1-\mathcal{H})\left(-\frac{db^*}{dI}\right) \leq \mathcal{H}\frac{df^*}{dI}\left(1-G(f^*)\right) \\ &\Longleftrightarrow \frac{(1-\mathcal{H})\left(-\frac{db^*}{dI}\right)}{\mathcal{H}\frac{df^*}{dI}} \leq \frac{1-G(f^*)}{1-G(b^*)}.\end{aligned}$$

Substituting (12) and (13) into the above equation, we get

$$\begin{aligned}\frac{d\pi_c}{dI} \leq 0 &\Longleftrightarrow \frac{\phi'(f^*)}{\phi'(b^*)} \leq \frac{1-G(f^*)}{1-G(b^*)} \\ &\Longleftrightarrow \frac{\phi'(f^*)}{1-G(f^*)} \leq \frac{\phi'(b^*)}{1-G(b^*)}.\end{aligned}$$

When $\zeta(x)$ is concave, $\phi(x) = x - \zeta(x)$ must be convex, thus $\phi'(x)$ is increasing in x. As a result, $\frac{\phi'(x)}{1-G(x)}$ must be increasing in x. We know that in equilibrium $f^* < b^*$, so we must have

$$\frac{\phi'(f^*)}{1-G(f^*)} < \frac{\phi'(b^*)}{1-G(b^*)}.$$

This implies $\frac{d\pi_c}{dI} < 0$. Therefore, if ζ is concave, when I decreases, overall customer welfare, π_c, increases. If ζ is convex, when I decreases, the change in overall customer welfare is ambiguous.

A.8 Proof of Proposition 8

We just need to show that $V(c_2 - dc) > V(c_2)$ for any $0 < c_2 < c_{NB}$, and $dc \ll 1$. Let

$$\begin{pmatrix} S_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} S^*(c_2) \\ f^*(c_2) \end{pmatrix}, V_2 = V(c_2);$$

and

$$\begin{pmatrix} S_1 \\ f_1 \end{pmatrix} = \begin{pmatrix} S^*(c_2 - dc) \\ f^*(c_2 - dc) \end{pmatrix}, V_1 = V(c_2 - dc).$$

The bank dealer's zero profit condition implies $S_2 < c_2$. Let $dc < c_2 - S_2$.

Starting with $c_B = c_2$, consider the case in which c_B decreases from c_2 to $c_2 - dc$. If the bank dealer still chooses the strategy (S_2, f_2), the zero profit condition will be violated and the bank dealer's profit will be positive. Suppose the bank dealer keeps $f = f_2$ unchanged, and decreases the spread S. When $S = f_2$, it is clear that the bank dealer's profit will be negative. By the mean value theorem, there exists $S_1' \in (f_2, S_2)$, such that the bank dealer's profit is zero. Overall customer welfare under (f_2, S_1') is strictly higher than that under (f_2, S_2), because all prices weakly decrease. Besides, the price menu (f_2, S_1') satisfies the zero profit condition with cost $c_B = c_2 - dc$. By definition of $V(c_2 - dc)$, overall customer welfare under (f_2, S_1') is weakly lower than $V(c_2 - dc)$. This implies

$$V(c_2 - dc) > V(c_2).$$

A.9 Proof of Proposition 9

In our extension with non-bank dealer matchmaking in Section 4.1, since $\mathcal{H}_{NB} < \mathcal{H}_B$, the bank dealer and the non-bank dealer provide differentiated matchmaking service, so both of them make positive profit even under price competition. The bank dealer's profit can be written as

$$\Pi_B = \frac{2\pi}{r}\left[\left(\mathcal{H}_B f_B - I\right)\frac{\frac{S-\mathcal{H}_B f_B}{1-\mathcal{H}_B} - z\left(f_B\right)}{A} + \left(S - c_B\right)\left(1 - \frac{\frac{S-\mathcal{H}_B f_B}{1-\mathcal{H}_B}}{A}\right)\right].$$

In the Section 4.1 extension,

$$z\left(f_B\right) = z_2\left(f_B\right) = \frac{\mathcal{H}_B f_B - \mathcal{H}_{NB} f_{NB}}{\Delta} = f_B + \frac{\mathcal{H}_{NB}}{\Delta}\left(f_B - f_{NB}\right).$$

while in our main model in Section 3, $z\left(f_B\right) = z_1\left(f_B\right) = f_B$. The bank dealer's FOC is

$$\frac{\partial \Pi_B}{\partial f_B} = \frac{2\pi}{rA}\left\{\left[\mathcal{H}_B\frac{S-\mathcal{H}_B f_B}{1-\mathcal{H}_B} - \left(\mathcal{H}_B f_B - I\right)\frac{\mathcal{H}_B}{1-\mathcal{H}_B} + \left(S - c_B\right)\frac{\mathcal{H}_B}{1-\mathcal{H}_B}\right] - \left[\mathcal{H}_B z\left(f_B\right) + \left(\mathcal{H}_B f_B - I\right)\frac{\partial z\left(f_B\right)}{\partial f_B}\right]\right\}.$$

In Section 4.1 we have $f_B > f_{NB}$ in equilibrium, thus

$$z_2\left(f_B\right) > z_1\left(f_B\right)$$

and

$$\frac{\partial z_2\left(f_B\right)}{\partial f_B} > \frac{\partial z_1\left(f_B\right)}{\partial f_B}.$$

In the constrained market-making equilibrium ($S = c_{NB}$), the equilibrium fee f_B is the solution of

$$\left.\frac{\partial \Pi_B}{\partial f_B}\right|_{S=c_{NB}} = 0.$$

For both $z_1\left(\cdot\right)$ and $z_2\left(\cdot\right)$, the first term of the bank dealer's FOC above

$$\mathcal{H}_B\frac{S-\mathcal{H}_B f_B}{1-\mathcal{H}_B} - \left(\mathcal{H}_B f_B - I\right)\frac{\mathcal{H}_B}{1-\mathcal{H}_B} + \left(S - c_B\right)\frac{\mathcal{H}_B}{1-\mathcal{H}_B}$$

is a linear, decreasing function of f_B, and the second term of the FOC

$$\mathcal{H}_B z\left(f_B\right) + \left(\mathcal{H}_B f_B - I\right)\frac{\partial z\left(f_B\right)}{\partial f_B}$$

is a linear, increasing function of f_B. Hence, the solution of

$$\left.\frac{\partial \Pi_B}{\partial f_B}\right|_{S=c_{NB}} = 0$$

in the Section 4.1 extension (f_B^2) and in the main model (f_B^1) satisfy

$$f_B^2 < f_B^1.$$